Improved Lossless Transform Coding of Audio Signals

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Abstract

Recent papers have proposed linear prediction as a useful method for lossless audio coding. Transform coding, however, has hardly been investigated, although it seems to be more suited for the harmonic structure of most audio signals. In this paper we present some results on lossless transform coding of CD-quality audio data. One main aspect lies on a convenient quantization method to guarantee perfect reconstruction. We achieve bit rates which are lower than those obtained by lossless linear prediction schemes.

1 Introduction

Lossless audio coding is a topic of high interest for both professional and customer applications. Modern lossy coding standards (e.g. ISO MPEG 1 and 2) can achieve large compression ratios with high subjective quality, however, multiple coding can reveal originally masked distortions. In addition, reproduction of critical music items shows that even the best systems can not be considered as truly transparent.

Applying entropy coding methods as Lempel-Ziv, Huffman or arithmetic coding directly to the audio signal is not very efficient due to the long-time correlations in a 16-bit 44.1 kHz signal. Therefore, conventional data compression tools fail in the case of digital audio data. A preprocessing stage, which eliminates the statistical dependencies within the signal, leads to an almost uncorrelated source which is easier to code. While there have been many papers which propose linear prediction for this preprocessing stage [1][2][3], the other usual tool for decorrelating signals, linear transforms, has hardly been investigated [5][6].

In this paper we discuss the application of discrete orthonormal transforms for lossless audio coding. First, a general coding scheme is presented to explain how the transform can be employed for lossless coding. Then some essential properties concerning the quantization of the transform coefficients are derived. Finally, we describe a practical coding algorithm and present some results.

2 Lossless Transform Coding

In general, the spectrum resulting from a signal transform will be real-valued, even for an integer input signal. For an efficient transmission the coefficients have to be quantized, which causes inevitable errors in the output signal. Therefore, lossless transform coding has to be considered as a combination of conventional lossy transform coding and additional transmission of the coding error. **Fig. 1** shows the coding scheme. The signal code c is derived from the input signal x using the lossy compression algorithm. To achieve lossless compression, the difference e between the input signal x and the reconstructed signal y is generated in the encoder by local decoding, and both the lossy compressed code c and the error e are transmitted. In the decoder, the error signal is added to the decoded approximation, resulting in an output signal which is a perfectly reconstructed version of the input signal.

It is obvious that the error signal depends on the lossy coding algorithm. If the compression ratio is high, the error signal will be large and correlated, and the coding problem passes from the original signal to the error signal. On the other hand, if compression is very small, the error will be zero, but not all of the signal's redundancy is removed. Thus, as a main goal in lossless transform coding, we have to find the best compromise between a high compression ratio in the lossy branch and an easily codeable signal in the correction branch.

We now consider a system as it is shown in **Fig. 2**. From the quantized input signal $x(n) \in \mathbf{Z}$ a set of transform coefficients $t(k)$ is calculated using an arbitrary orthonormal transform $\mathbf{A}$ with block length N. The coefficients are scaled by α and quantized with an unitary quantization step size $\Delta = 1$, leading to an integer-valued spectrum $c(k)$ which - after entropy coding - is finally transmitted. Using a suitable transform, many of the coefficients are very small or even zero, and, moreover, they constitute an uncorrelated source. Hence, the integer-valued spectrum can be easily entropy-coded without taking into account joint probabilities.

As a result of the quantization, which is equivalent to integer rounding, decoding of $c(k)$ does not guarantee perfect reconstruction, although the integer spectrum itself is coded losslessly. Therefore, we have to check for possible errors by decoding $c(k)$ in the encoder and to generate an error correction signal, if necessary.

After descaling with α^{-1} and applying the inverse transform $\mathbf{A}^{-1} = \mathbf{A}^{t}$, where $\mathbf{A}^{t}$ stands for the transposed matrix, we obtain the real-valued signal $y'(n)$ which, due to the quantization in the transform domain, is not the original $x(n)$. However, since we consider integer signals, there is no need to reconstruct the input signal exactly by a real-valued signal. If $|y'(n) - x(n)| < 0.5$, integer rounding of $y'(n)$ leads to a reconstructed integer signal $y(n)$ which is identical with $x(n)$. Otherwise, the input signal is not perfectly reconstructed, i.e. we have an error $e(n) \neq 0$. Of course, $e(n) = x(n) - y(n)$ is an integer signal, because $x(n)$ and $y(n)$ are integer as well. The error signal $e(n)$ has to be transmitted in addition to the coefficients $c(k)$.

On the decoder side, $y(n)$ is calculated identically by descaling, inverse transformation and integer rounding. After adding $e(n)$, we get $y(n) + e(n) = y(n) + (x(n) - y(n)) = x(n)$, which is the desired original input signal.

3 Quantization effects

3.1 Theoretical bounds

Obviously, the scalefactor α has great influence not only on the coefficients $c(k)$, but also on the error signal $e(n)$. Of course, there is also a relation between the error bit rate R_e and the bit rate R_c for the coefficients. Let R_0 denote the coefficient bit rate for the case $\alpha = 1$, then R_c rises with $\mathrm{ld}(\alpha)$:

$$R_c = R_0 + \mathrm{ld}(\alpha).$$

To derive a bound for the error bit rate, the process of scaling, integer rounding and descaling is considered as a linear quantization with a new step size $\Delta = 1 / \alpha$, leading to an equally distributed quantization error $q(k) = t(k) - t'(k)$ in the transform domain. Thus, the first order entropy of this quantization error is simply $H_1(q) = \mathrm{ld}(\Delta) = -\mathrm{ld}(\alpha)$.

The unquantized error $\varepsilon(n) = x(n) - y'(n)$ in the time domain has the same variance $\sigma_\varepsilon^2 = \sigma_q^2$ due to the energy conservation property of any orthonormal transform. Since for a large transform length N the error $\varepsilon(n)$ can be seen as a superposition of many equally distributed random variables, the assumption of a gaussian probability density function (PDF) for $\varepsilon(n)$ is justified. For a large step size Δ, i.e. for $\alpha << 1$, the quantized error $e(n)$ has also a (discrete) gaussian PDF with same variance, so its entropy $H_1(e)$ is about $0.5\cdot\mathrm{ld}(\pi e/6) \approx 0.255$ bit higher than $H_1(q)$, as shown in [4].

Hence, for $\alpha << 1$, R_e is bounded by

$$R_e \geq 0.255 - \mathrm{ld}(\alpha),$$

assuming an uncorrelated error signal. The overall bit rate is $R = R_c + R_e$, and substitution of R_c and R_e leads to

$$R \geq R_0 + 0.255.$$

Thus, the overall bit rate is at least 0.255 bit higher than the coefficient bit rate for $\alpha = 1$. These results can be practically verified, as it is shown in **Fig. 3**. Generally, the bit rate R_e will be higher than $H_1(e)$, but we assume that with a suitable entropy code this bound can be approximated, so the sum $R_c + H_1(e)$ is a good approximation for the achievable overall bit rate R. It can be seen that for $\alpha < 1$ we obtain a nearly constant overall bit rate, which is slightly higher than R_0, whereas for $\alpha > 1$, the overall bit rate increases with $\mathrm{ld}(\alpha)$.

3.2 Error probabilities

We already mentioned that the unquantized error $\varepsilon(n)$ in the time domain has a gaussian PDF

$$f(\varepsilon) = \frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}} \cdot e^{-\frac{\varepsilon^2}{2\sigma_\varepsilon^2}}$$

with the variance of the equally distributed quantization error

$$\sigma_\varepsilon^2 = \sigma_q^2 = \frac{\Delta^2}{12} = \frac{1}{12\alpha^2}.$$

Since we consider integer input signals, only $|\varepsilon(n)| \geq 0.5$ leads to an error $e(n) \neq 0$. Therefore, the probability for the occurrence of an error in the reconstructed signal $y(n)$ is

$$p_e = P\{|\varepsilon| \geq 0.5\} = 2 \cdot \int_{0.5}^{\infty} f(\varepsilon)\,\mathrm{d}\,\varepsilon = \mathrm{erfc}\left(\alpha\sqrt{3/2}\right) .$$

The error probability decreases with a growing scalefactor, but according to the above equation it will never be zero, not even for a very large α. This is due to the fact that the assumption of a gaussian PDF for the error signal is theoretically correct only for $N \to \infty$. Actually, if the scalefactor is very large, the error probability approaches zero. But from **Fig. 3** it is obvious that a large value of α is not feasible for practical applications, because it leads to a very high overall bit rate. Since the overall bit rate typically reaches its minimum for $\mathrm{ld}(\alpha) \leq -2$ (see **Fig. 3**), we have chosen $\alpha = 0.25$.

4 Coding Algorithm

A simplified block diagram of the implemented lossless transform coding system is shown in **Fig. 4**. The coder uses an orthonormal DCT with either fixed or variable block length. Each integer-valued spectrum $c(k)$ is divided into groups of 32 adjacent coefficients. This partition proved to be most efficient. Since these groups have an almost laplacian PDF, the codebook consists of several Rice codes [1]. Each group is coded using the most convenient Rice code, i.e. the code which leads to a minimum number of bits. A Rice code is in fact a Huffman code for a laplacian PDF, which is determined by its standard deviation σ. Since Rice codes only exist for discrete values of σ, only the indices of the chosen codes have to be transmitted.

The inverse transform allows for the generation of the error signal, which is encoded using an arithmetic coder with a static model.

In fixed block length mode, each block of N input samples is transformed using a DCT with the same length. In adaptive block length mode, the input signal $x(n)$ is divided into blocks of M samples. Each combination of M-, $M/2$- and $M/4$-point transforms is calculated, and for the corresponding block the most suitable combination is finally selected.

The decoder applies the inverse transform after decoding of the coefficients. By adding the decoded error signal, the original signal is perfectly reconstructed. The computational complexity of the decoder is about half of that of the encoder, because only one transformation has to be performed.

5 Coding results

In general, a higher block length leads to better coding results due to a better decorrelation of the source signal, except for signals with very fast varying statistics, pitched signals like speech or very transient signals like castanets. As a drawback, editing becomes more difficult for higher block lengths. The adaptive block length mode only slightly improves the results compared to the best fixed block length for each individual signal. However, this feature is helpful, since the choice of an appropriate fixed block length sometimes turns out to be difficult.

In **Table 1**, the results for our Lossless Transform Audio Compression (LTAC) algorithm with fixed and adaptive block length are compared to those obtained by linear predictive coding, namely the popular program "Shorten" [7], used with default parameters (polynomial prediction) combined with the -c 2 option for stereo files.

Category	Shorten	LTAC (fixed)	LTAC (adaptive)
Alignment signals	6,29	6,29	6,06
Artificial signals	2,84	2,92	2,60
Single instruments	4,60	4,28	4,15
Vocal	5,35	4,91	4,83
Speech	5,45	5,49	5,36
Solo instruments	5,09	4,65	4,52
Vocal & Orchestra	6,73	6,22	6,14
Orchestra	5,33	5,14	5,07
Pop Music	6,37	6,14	6,03
Total	**4,98**	**4,69**	**4,56**

Table 1: Coding results (bits per sample) for the SQAM disc [8]. The categories are based on the according SQAM sections. Fixed block length: N = 2048, adaptive block length: M = 4096. LTAC version 1.61.

For most categories, the results of LTAC are significantly better than those of Shorten. Other lossless audio coders based on linear prediction are able to achieve 4.83 bps [1] and 4.68 bps [3] for the whole SQAM disc.

6 Conclusions

We have presented a lossless audio coding algorithm which is based on orthonormal signal transforms, unlike previous algorithms for lossless audio coding which mainly used linear prediction. We have shown how perfect reconstruction of the 16-bit input signal can be achieved using integer transform coefficients and an additional integer error signal. The average bit rates are below those obtained by common linear prediction schemes.

Precompiled versions of LTAC are available for Windows 95/98/NT, DOS and Solaris at *http://www-ft.ee.tu-berlin.de/~liebchen/ltac.html*.

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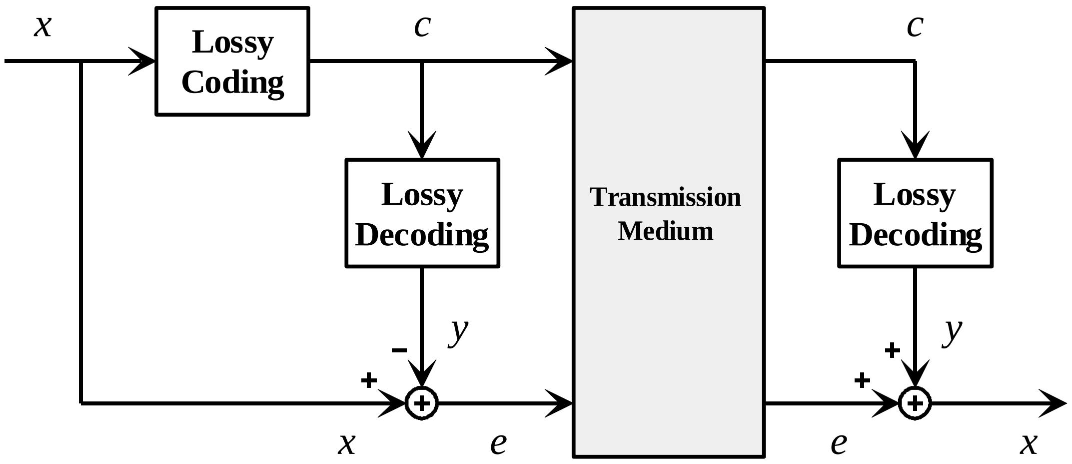

Fig. 1: Lossless coding as a combination of lossy coding and additional error transmission.

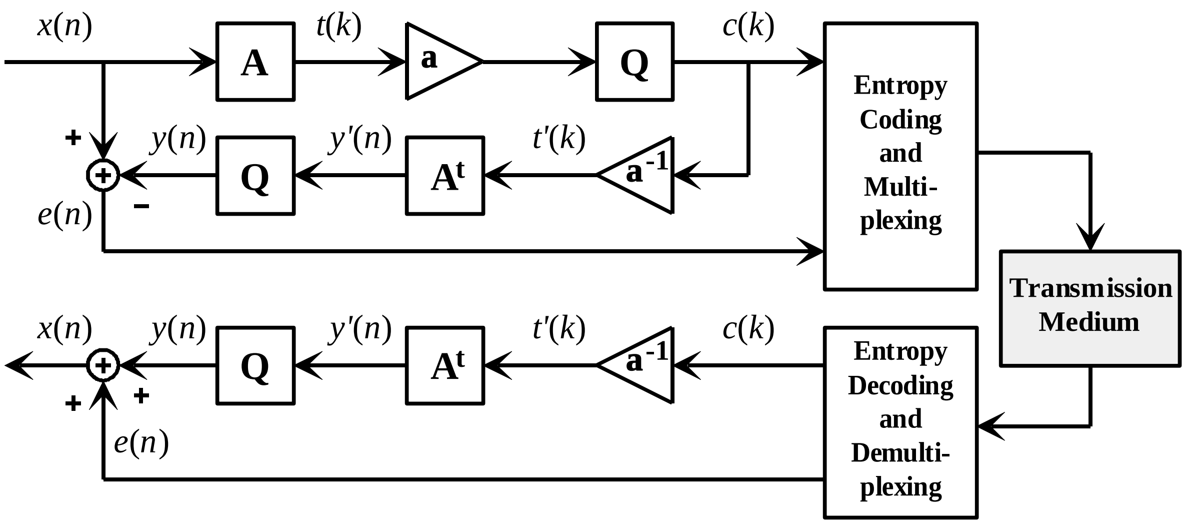

Fig. 2: Block diagram of a lossless transform coding system. Q: Quantization with step size $\Delta = 1$.

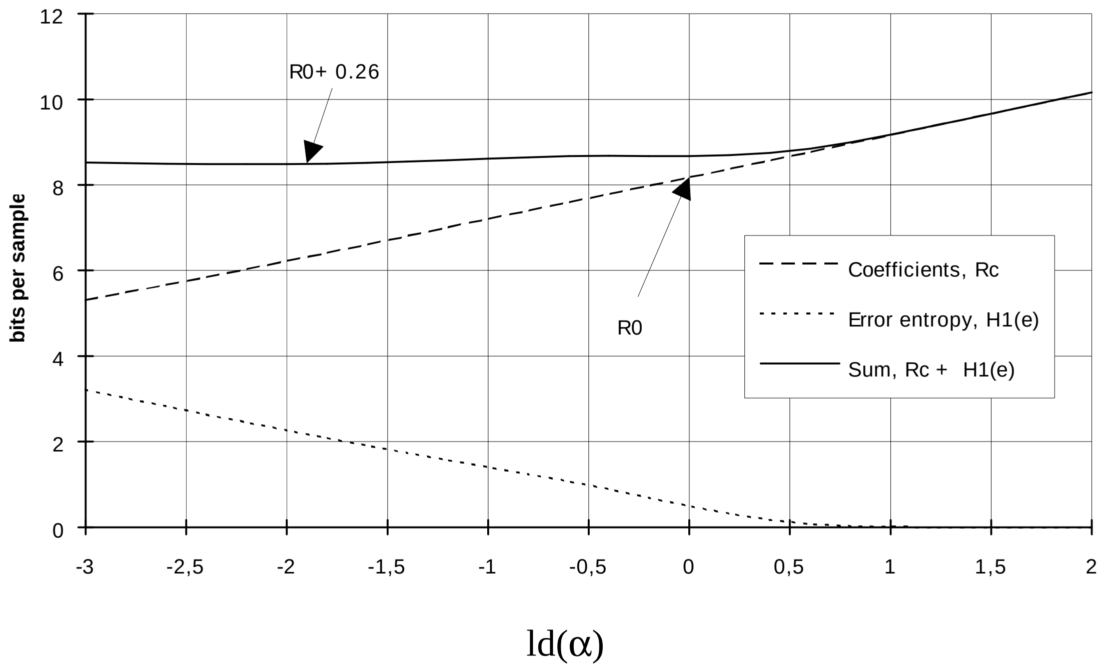

Fig. 3: Bit rates depending on the scalefactor. The values have been measured for a piece of pop music.

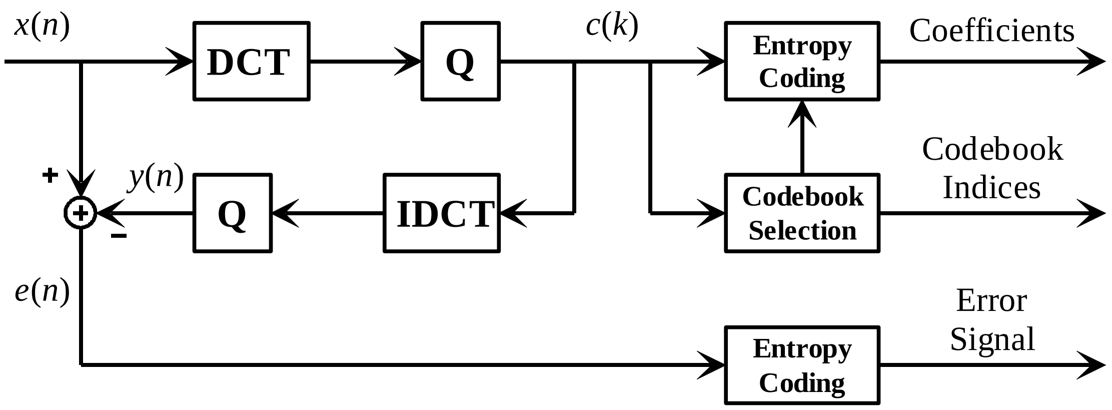

Fig. 4: Simplified block diagram of the implemented lossless transform coding system. Q: Quantization with step size $\Delta = 1 / \alpha$.