Color Image Noise Reduction using Perceptual Maximum Variation Modeling for Color Diffusion

Jangheon Kim and Thomas Sikora

Department of Communication Sytems
Technical University of Berlin, Einsteinufer 17, 10587 Berlin, Germany
{*j.kim, sikora*}*@nue.tu-berlin.de*

Abstract Diffusion is an efficient localized image regularization method based on the analysis of image structures such as direction and magnitude. However, the localization at weak features which have small brightness variations is fundamentally difficult. This often results in removal of weak features. We address this problem with perceptual maximum variation modeling. In our method, diffusion flow of color images is performed by evaluating the perceptual maximum variations which combine the small differences in both brightness and chromaticity, using a least squares optimization with principal component analysis (PCA). A consistency constraint is employed to avoid influence from global color distributions and to enhance homogeneous color regions. We apply our approach for de-noising of color images and obtain excellent improvements over existing methods.

1 Introduction

Removal of noise in color images is an important task for many applications. Here it is of vital importance to reduce noise while preserving important image structures such as strong and weak edges and fine texture details. Scalar diffusion theory has been utilized for this purpose in the past [1]. The method employs regularization of images usually using a fluid mechanic basis which equilibrates spatial variations in concentration. However, regularization often results in over-diffusion i.e. delocalized flow and thus in blur of small brightness variations in images. If the important edges are formed by small brightness variations, over-diffusion causes blurring problem that removes the edges. Recent diffusion methods handles the problem using tensor-valued i.e. vector diffusivity function [6] which can be adapted to local edge orientation. The function allows smoothing along the edges, but not perpendicular to it. However, the method is just suitable for a grey image or one channel processing. An independent process of each color channels results color distortions due to subtleties of color correlation between the channels [7, 8]. Although an edge between head and body of a colorful bird in Figure 1 can be clearly distinguished in the red channel, diffusion error may occur in the other channels.

(a) (b) (c) (d)

Fig. 1 (a) colorful bird image and the subtleties of correlation between (b) red, (c) green, and (d) blue channels.

Some studies of color filtering deal with the channel mixture problem using color vectorial methods - thus extending the well established scalar diffusion. An efficient method is to extend the medians of the scalar space to the color vectorial data [2 - 5]. Vector median filters (VMF) [2, 3] are obtained by considering L1, L2 norms for ordering image vectors according to their relative magnitude differences. Other method consider vector directional filters (VDF) [4, 5]. These works are related with heuristic approach which makes homogeneity directed correlation among the color channels. However, the localization performance on ambiguous edges with weak variations is still unstable. Human visual system perceives small brightness variations using a knowledge-based analogy from color components. Perceptually motivated color spaces are used to evaluate mutual coherency and geometrical continuation [8]. Although these methods achieve good results for preserving colors, they do not preserve small brightness variations in the color images.

In this paper we introduce perceptual maximum variation modeling which is optimized to combine the small differences in both brightness and chromaticity. A consistency constraint for the modeling is considered to avoid influence from global color distributions and to enhance homogeneous color regions. We apply our approach for de-noising of color images with anisotropic diffusivity function.

2 Robust Color Diffusion using Perceptual Maximum Variation Modeling

The basic idea is based on the fact that the human visual system detects the edges of color regions by considering chromaticity and brightness simultaneously. If the brightness difference of neighboring regions is very small, our eyes can detect the important edges using chromaticity difference between regions. We estimate a

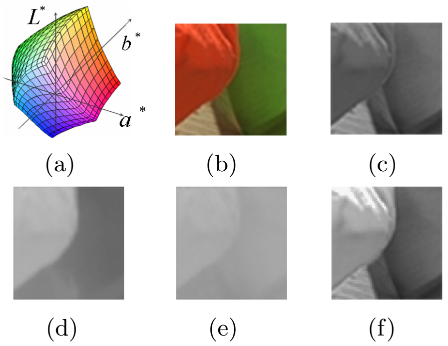

Fig. 2 (a) CIE-$L^*a^*b^*$ color space (b) original color image (c) L^* (d) a^* and (e) b^* of the color image, (f) $\wp$-space from our modeling (here, the scale of $\wp$-space is 255 for visualization purpose).

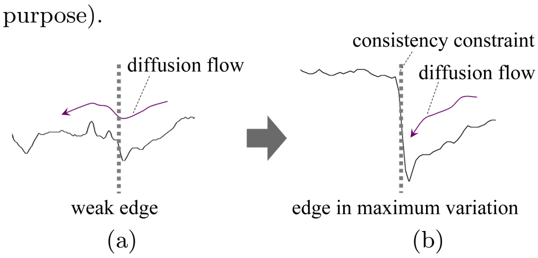

Fig. 3 (a) localization error of diffusion on a weak edge (i.e. over-diffusion artifacts) (b) our method.

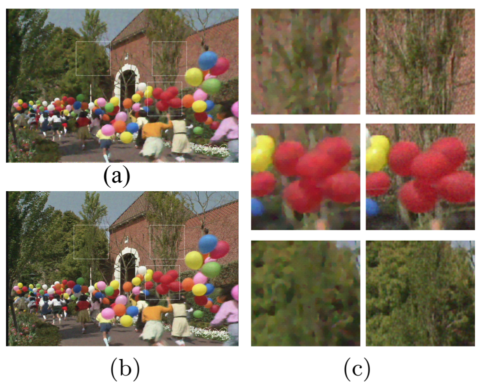

Fig. 4 (a) Perona and Malik color diffusion for 10% Gaussian noise image, PSNR=26.199dB (b) Proposed method for 10% Gaussian noise image, PSNR=33.518dB (c) details with over-diffusion on weak edges (left: 4a, right: 4b).

higher dimensional perceptual $\wp$-space (note that the scale of $\wp$-space can be scaled over the size of image colors) projecting the maximum variations of both brightness and chromaticity. First, a color image is converted into a perceptually uniform color space of CIE-$L^*a^*b^*$, as depicted in Figure 2. The small brightness variations on a color edge are selectively modulated in luminance axis to preserve differences of chromaticity. PCA is used to find a set of orthogonal vectors lying along the direction of the maximal variation. The modulated variations can be projected into a perceptual dimension which has high dynamic range. During the optimization, the consistency constraint is considered to enhance the localization of homogeneity between color regions by removing the diversity of global color distribution from the image. Accordingly, color diffusion evaluating the maximum variations allows excellent localization performance. This is illustrated in Figure 3. Figure 4 shows that in this way ill-posed over-diffusion of traditional diffusion methods can be overcome when the color diffusion is performed by evaluating variations of $\wp$-space.

2.1 Perceptual maximum variation modeling

Perceptual difference between two neighboring color pels in CIE-$L^*a^*b^*$ can be simply measured by the Euclidean distance between the two vectorial values $c_p(\mathrm{x}) = [L_p^*, a_p^*, b_p^*]$ and $c_q(\mathrm{x}) = [L_q^*, a_q^*, b_q^*]$ with perceptual color metric δc_{pq}^* in (1). x denotes a pel $\mathrm{x} = (x, y)$ on a CIE-$L^*a^*b^*$ color domain $I(\mathrm{x}) : \mathbb{R}_+^3 = \{c(L_p^*, a_p^*, b_p^*) \geqslant 0\}$.

$$\delta c_{pq}^* = \sqrt{\left(L_p^* - L_q^*\right)^2 + \left(a_p^* - a_q^*\right)^2 + \left(b_p^* - b_q^*\right)^2} \qquad (1)$$

Colors with the same δc_{pq}^* are perceptually equal. The brightness $\omega(\mathrm{x})$ in luminance domain L^* and the chromaticity $\xi(\mathrm{x})$ includes a^* and b^* respectively. It represents the length of the color vectors and the normalized color components.

$$\omega(\mathrm{x}) = \sum_{i=1}^{n} \|I_n(\mathrm{x})\| \;\; and \;\; \xi(\mathrm{x}) = I(\mathrm{x})/\omega(\mathrm{x}) \qquad (2)$$

We estimate visually maximum variations using the perceptual difference which combines $\omega(\mathrm{x})$ and $\xi(\mathrm{x})$ in color metric δc_{pq}^*. A linear transform which projects original brightness in the luminance L_p^* and L_q^* into perceptual intensity $\wp_p$ and $\wp_q$ (which remains proportional to the perceptual color difference) is obtained by minimizing the following quadratic function. The constant K is chosen for the proportional weight e.g. when $K = 1$, L_p^* and L_q^* are equally modulated with the perceptual differences.

$$\wp\left(c_{0,\cdots,p,q,\cdots,n}(\mathrm{x})\right) = \sum_{p=0}^{n-2} \sum_{q=p+1}^{n-1} \left(|\wp_p - \wp_q|/\delta c_{pq}^* - K\right)^2 \qquad (3)$$

$\wp_p$ and $\wp_q$ generally have higher dynamic range than L_p^* and L_q^* , since they include total difference of both brightness and chromaticity. $\wp$-space has maximum variations in human visual range. Using PCA equation (3) the entire distribution of color values can be projected onto the primary L^*-axis of the ellipsoid using a linear transform. The principal components are the eigenvectors of its covariance matrices. By calculating the covariance matrix with largest eigenvalue the perceptual maximum variation can be estimated.

2.2 Consistency constraint

PCA performs very well in aligning the colors in a region which have locally-compact or globally-smooth color distributions. If we deal with whole images in a noisy condition, the color whole image distribution may not be suitable to fit local properties. Equation (3) is convex, but may converge into multiple global minima. We solve the problem by considering a consistency constraint. First, the chromaticity difference between two color pels is defined by a equation similar to (1) as

$$\delta\xi_{pq} = \sqrt{(a_p^* - a_q^*)^2 + (b_p^* - b_q^*)^2} \tag{4}$$

As shown in Figure 2d and Figure 2e, the chromaticity domain can be used as a good consistency measure because it inherently has piecewise smoothness over the image - while the brightness in L^* is affected by saturation, lightness e.g. shadow, reflection and noise, etc. The consistency measure is decreased when the spatial distance to the pel under consideration increases due to the coherency of objects. In this paper, consistency Λ is defined as a similarity group based on probability, using the chromaticity difference ξ_{pq} and spatial distance d_{pq} with proportional constant k. γ_ξ and γ_d are empirically determined in (5).

$$\Lambda\left(c_{0,\cdots,p,q,\cdots,n}(\mathrm{x})\right) = \sum_{i=1}^{n} k \cdot exp\left(-(\delta\xi_{pq}/\gamma_\xi + d_{pq}/\gamma_d)\right) \tag{5}$$

2.3 Robust color diffusion with perceptual maximum variation modeling

If a noisy color image is considered as a noisy color vector space $I(\mathrm{x}) : \mathbb{R}^2 \to \mathbb{R}^n$, the space can be separated into brightness $\omega(\mathrm{x}) : \mathbb{R}^2 \to \mathbb{R}^+$ and chromaticity $\xi(\mathrm{x}) : \mathbb{R}^2 \to S^{n-1}$. We deal with the brightness and chromaticity components separately for diffusion. First, the diffusion of brightness is calculated by tensor-valued diffusivity function

$$\partial_t\omega(\mathrm{x}) - div(D \cdot \nabla\omega(\mathrm{x})) = 0 \tag{6}$$

D denotes a positive definite symmetric matrix called "diffusion tensor" and ∂_t is the derivation with respect to the time. Instead of evaluating the gradient of initial brightness, we propose to obtain a perceptual diffusion flow tensor using the Cartesian product of the gradient vector $(Q_x, Q_y)^T$ [6] in the $\wp$-space resulting from (3) with itself. While the small variations $\nabla\omega(\mathrm{x})$ of brightness are well filtered, important structure in the variations is still preserved by localized diffusion flow in the perceptual maximum.

$$D\left(\nabla\wp\right) = \begin{pmatrix} Q_x^2 & Q_xQ_y \\ Q_xQ_y & Q_y^2 \end{pmatrix} \tag{7}$$

$Q_x = \wp(\mathrm{x}) * G_{x,\sigma}$ and $Q_y = \wp(\mathrm{x}) * G_{y,\sigma}$ are obtained by x- and y-directional derivatives of 2D Gaussian kernel $G(\mathrm{x}) = (2\pi\sigma^2)^{-1}\exp\left(-(x^2+y^2)/2\sigma^2\right)$ with a standard deviation σ. With a scale σ of successively smoothed concentrations, its eigenvectors describe the direction of highest and lowest contrast. These contrasts are given by corresponding two positive eigenvalues. Anisotropic diffusion of brightness can be considered by an edge-stopping weighting function (e.g. Perona-Malik g-weight, and Tukey bi-weight, etc. in [1]) of the eigenvalues.
Let the chromaticity $\xi_i(\mathrm{x})$:$\mathbb{R}^2 \to \mathbb{R}$ describes each of the n-components of $\xi(\mathrm{x})$. The gradients of the components $\nabla\xi_\mathrm{i} = (\partial\xi_i/\partial x)\mathbf{x} + (\partial\xi_i/\partial y)\mathbf{y}$ can be defined with unit vectors $\mathbf{x}$ and $\mathbf{y}$ which have the values of the component gradients $\|\nabla\xi_\mathrm{i}\| = \left((\partial\xi_i/\partial x)^2 + (\partial\xi_i/\partial y)^2\right)^{1/2}$ in the x- and y-directions. We solve a constrained minimization problem called "harmonic map" [7] with a constraint $\|\nabla\xi\| = 1$ as

$$\partial_t\xi_i - div(\|\nabla\xi\|^{p-2} \cdot \nabla\xi_i) + \xi_i\|\nabla\xi\|^p = 0,\ 1 \leqslant i \leqslant n \tag{8}$$

Here, $\|\nabla\xi\| = \sum_{i=1}^{n}((\partial\xi_i/\partial x + \partial\xi_i/\partial y)^{1/2}$ is the absolute value of the image gradient, i.e. total component gradient. When $p = 2$, (8) equals to the isotropic diffusion $\partial_t\xi - \Delta\xi_i + \xi_i\|\nabla\xi\|^p = 0$, which substitutes the divergence term in (8) into a component Laplacian $\Delta\xi_i = \left(\partial^2\xi_i/\partial x^2 + \partial^2\xi_i/\partial y^2\right)^{1/2}$. Anisotropic diffusion in chromaticity results in the range of $1 \leqslant i \leqslant 2$ given in the weighting function.

3 Experimental Results

In our experiments we investigated the efficiency of our proposed method on zero-mean Gaussian color noise and zero-mean impulsive color noise. Impulse color noise had random amplitude and spectral content with a large perturbation of the color values. Figure 5 shows results of our method using Balloons and Parrot images with 10% Gaussian color noise for each color channel in original color image. Perceptual modeling estimates maximum variations in color consistency to enhance convergence. Then, diffusion process is less sensitive to momentary variations e.g. noise but more sensitive to discontinous variations e.g. edges between two regions. In the geometric continuation, robust de-noising is achieved even in image regions containing weak edges and small brightness variations by iterative diffusion with a scale. PSNR$[dB]$ is evaluated for color in three channels as

$$10\log\sum\nolimits_{i=1}^{N_1}\sum\nolimits_{j=1}^{N_2}\left(255 \cdot 3N_1N_2/\left\|I(i,j) - I'(i,j)\right\|^2\right) \tag{9}$$

Table 1 Comparison using Lena image with 4, 10% color impulsive noise

Algorithms	4%	10%
None	17.983	11.702
Arithmetic mean filter 3x3(AMF)	25.971	14.802
Perona and Malik color(PM) [1]	28.146	17.508
Vector directional filter(VDF) [2]	30.466	17.716
Vector median filter(VMF) [4]	31.427	22.342
Fuzzy vector directional filter(FVDF) [3]	30.827	20.089
Modified vector median filter(MVMF) [5]	38.446	26.411
Proposed method	39.841	27.834

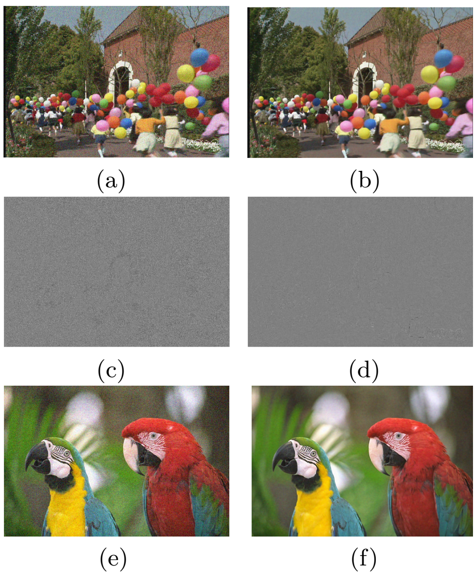

Fig. 5 (a) Balloons image with 10% Gaussian noise (b) de-noising result of 5a using our method, PSNR=33.518dB (c) error image of 5a (d) error image of 5b (e) Parrot image with 10% Gaussian noise (f) de-noising from 5e, PSNR=38.146dB.

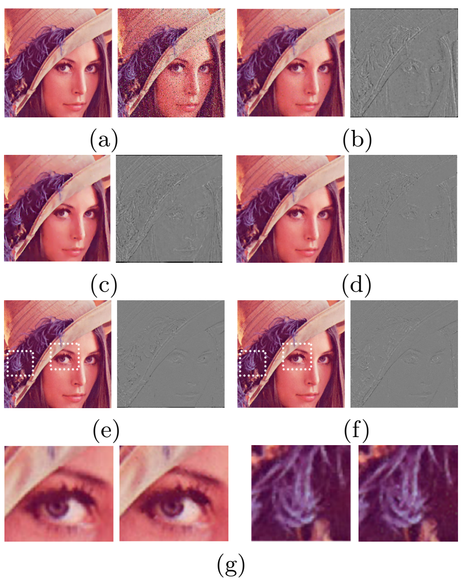

Fig. 6 De-noising effect of Table 1 - (a) original and distorted image with 4% impulse noise (b) AMF (c) PM (d) VMF (e) MVMF (f) proposed method (g) differences between 6e and 6f.

Table 1 depicts the efficiency of the proposed method compared with heuristic correlated color filtering approaches i.e. AMF, VMF, FVMF, VDF and MVDF for 4% and 10% percentages of color impulsive random noise. In [5], comparison of more methods is given. The comparisons verifies the performance of spatio adaptation. The proposed method with perceptual maximum variation modeling achieved significant improvements over the other techniques, ranging from 1.5 dB to 13.9 dB in PSNR. Figure 6 is provided for visual comparison of results. Figure 6g shows that our method results in sharper and more detailed structures after de-noising.

4 Summary and Conclusion

In this paper, we proposed a robust color image noise reduction method using perceptual maximum variation modeling. The maximum variations of brightness that is easily visible by a distortion is modelled by smoothly varing chromaticity. A combination of geometric continuations in both brightness and chromaticity localizes diffusive filtering. This method yield an excellent color denoising performance preserving image structures.

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